20. *(a)* Given

$$y = 10 + 2x_1^2 x_2 + 3x_2^2 x_3^2$$

Find all second-order partial derivatives. 4

(b) Given

$$y = x_1^2 + 2x_2 \text{ where } x_2 = x_1^3 + 5$$

Find out total derivative $\frac{dy}{dx_1}$. 2

(c) If the utility function is

$$u = \log(ax_1 + bx_2 + c\sqrt{x_1 x_2})$$

obtain the ratio of marginal utility. 4

21. *(a)* Given a demand function in implicit form

$$F(Q_1, P_1, P_2, Y) = 10P_1Q_1 + 5Q_1 - 2P_2 - 4Y - 18 = 0$$

Use implicit function rule to find—

(i) own price elasticity, E_{11};

(ii) cross-price elasticity, E_{12};

(iii) income elasticity at a point $(P_1, P_2, Y) = (2, 1, 20)$. 5

(b) Given the function $Q = AK^{\alpha}L^{\beta}$, where A, α, β are parameters and positive.

(i) Show that the function is a linear homogeneous function. 2

(ii) Prove that

$$K\frac{\partial Q}{\partial K} + L\frac{\partial Q}{\partial L} = Q$$

when the function is linearly homogeneous. 3

22. *(a)* Mention the characteristics of convex function with more than one explanatory variable. 3

(b) Derive the first- and second-order conditions in order to show that indifference curve is negatively sloped and convex to the origin taking the utility function $u = f(x, y)$, where u = total utility, x and y are the quantities of two commodities. 7

23. *(a)* Find the extreme values of the function

$$y = 4x_1^3 + 8x_1x_2 - 4x_1^2 - x_2^2 + 10$$ 5

(b) Given the utility function $u = 2xy$ subject to the budget constraint $3x + 4y = 90$. Find out the equilibrium values of x and y that maximize total utility. 5

17. *(a)* Given

$$Q_d = 10 - 5P$$
$$Q_s = -10 + 5P$$
$$\frac{dp}{dr} = 3(Q_d - Q_s)$$

Analyze the market model for stability. 4

(b) Distinguish between homogeneous and non-homogeneous equations. 3

(c) Write the method of solution of second-order linear differential equation. 3

18. *(a)* Given

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Show that $|A| = |A'|$. 3

(b) If

$$\begin{vmatrix} k-2 & 1 \\ 5 & k+2 \end{vmatrix} = 0$$

then find k. 2

(c) Solve the following system of simultaneous equations by matrix inversion : 5

$$2x_1 + 3x_2 - x_3 = 15$$
$$4x_2 + 2x_3 = 16$$
$$3x_1 + 2x_2 = 18$$

19. *(a)* Solve the linear market model : 5

$$Q_d = 50 - 2P$$
$$Q_s = -10 + 3P$$
$$Q_d = Q_s$$

(b) In a two-economy model denoted by the subscripts 1 and 2

$$C_1 = 0 \cdot 8Y_1$$
$$M_1 = 0 \cdot 2Y_1$$
$$Y_1 = C_1 + 200 + (X_1 - M_1)$$
$$X_1 = M_2$$

$$C_2 = 0 \cdot 7Y_2$$
$$M_2 = 0 \cdot 15Y_2$$
$$Y_2 = C_2 + 100 + (X_2 - M_2)$$
$$X_2 = M_1$$

find the equilibrium national incomes Y_1 and Y_2 using matrix algebra. 5

2022/TDC(CBCS)/EVEN/SEM/ECOHCC–202T/502

TDC (CBCS) Even Semester Exam., 2022

ECONOMICS

(Honours)

(2nd Semester)

Course No. : ECOHCC–202T

(Mathematical Methods in Economics—II)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* questions : 2×10=20

1. Define first-order linear differential equations.

2. Solve

$$\frac{dy}{dx} + 2x = 0$$

with initial condition $y(0) = 5$.

3. Define an exact differential equation.

4. Write two differences between scalar product and vector product.

5. What is orthogonal matrix? Give one example.

6. What is total differential?

7. Mention two characteristics of homogeneous function.

8. Given

$$y = 3x_1 + \frac{x_1}{x_2} + 10x_2^2$$

Find $\frac{\partial y}{\partial x_1}$.

9. Mention two characteristics of convex function.

10. Mention the geometric definition of concavity and convexity for a two-variable function $z = f(x_1, x_2)$.

11. What is unconstrained optimization?

12. What is technology coefficient matrix?

13. Write the economic meaning of

$$\sum_{i=1}^{n} a_{ij} < 1$$

in Leontief static open model.

14. Write two assumptions of technology coefficient matrix (*A*) in Leontief static open model.

15. Mention two limitations of Leontief static open input-output model.

Section—B

Answer any *five* questions : 10×5=50

16. (a) Find the solution of the following differential equation : 4

$$\frac{dy}{dx} + 3x^2y = 3x^2$$

(b) Solve : 3

$$3\frac{dy}{dt} + 6y = 5; \quad y(0) = 0$$

(c) Write down the general procedure of solving exact differential equation. 3